

PARI-GP Reference Card

(PARI-GP version 2.5.0)

Note: optional arguments are surrounded by braces {}.

Starting & Stopping GP

to enter GP, just type its name: `gp`
to exit GP, type `\q` or `quit`

Help

describe function `?function`
extended description `??keyword`
list of relevant help topics `???pattern`

Input/Output & Defaults

output previous line, the lines before `%, %', %'', etc.`
output from line n `%n`
separate multiple statements on line `;`
extend statement on additional lines `\`
extend statements on several lines `{seq1; seq2};`
comment `/* ... */`
one-line comment, rest of line ignored `\ \ ...`
set default d to val `default({d}, {val}, {flag})`
mimic behavior of GP 1.39 `default(compatible,3)`

Metacommands

toggle timer on/off `#`
print time for last result `##`
print $%n$ in raw format `\a n`
print defaults `\d`
set debug level to n `\g n`
set memory debug level to n `\gm n`
enable/disable logfile `\l {filename}`
print $%n$ in pretty matrix format `\m`
set output mode (raw=0, default=1) `\o n`
set n significant digits `\p n`
set n terms in series `\ps n`
quit GP `\q`
print the list of PARI types `\t`
print the list of user-defined functions `\u`
read file into GP `\r filename`
write $%n$ to file `\w n filename`

GP Within Emacs

to enter GP from within Emacs: `M-x gp, C-u M-x gp`
word completion `(TAB)`
help menu window `M-\c`
describe function `M-?`
display T_EX'd PARI manual `M-x gpman`
set prompt string `M-\p`
break line at column 100, insert `M-\l`
PARI metacommand `\letter` `M-\letter`

Reserved Variable Names

$\pi = 3.14159\dots$ `Pi`
Euler's constant $= .57721\dots$ `Euler`
square root of -1 `I`
big-oh notation `O`

PARI Types & Input Formats

`t_INT/t_REAL`. Integers, Reals $\pm n, \pm n.ddd$
`t_INTMOD`. Integers modulo m `Mod(n, m)`
`t_FRAC`. Rational Numbers n/m
`t_FFELT`. Elt in a Finite Field `ffgen(T)`
`t_COMPLEX`. Complex Numbers $x + y * I$
`t_PADIC`. p -adic Numbers $x + O(p^k)$
`t_QUAD`. Quadratic Numbers $x + y * \text{quadgen}(D)$
`t_POLMOD`. Polynomials modulo g `Mod(f, g)`
`t_POL`. Polynomials $a * x^n + \dots + b$
`t_SER`. Power Series $f + O(x^k)$
`t_QFI/t_QFR`. Imag/Real bin. quad. forms `Qfb(a, b, c, {d})`
`t_RFRAC`. Rational Functions f/g
`t_VEC/t_COL`. Row/Column Vectors $[x, y, z], [x, y, z]~$
`t_MAT`. Matrices $[x, y, z; t; u, v]$
`t_LIST`. Lists `List([x, y, z])`
`t_STR`. Strings `"aaa"`

Standard Operators

basic operations $+, -, *, /, ^$
`i=i+1, i=i-1, i=i*j, ...` $i++, i--, i*=j, \dots$
euclidean quotient, remainder $x \backslash y, x \backslash y, x \% y, \text{divrem}(x, y)$
shift x left or right n bits $x << n, x >> n$ or `shift(x, $\pm n$)`
comparison operators `<=, <, >=, >, ==, !=`
boolean operators (or, and, not) `||, &&, !`
sign of $x = -1, 0, 1$ `sign(x)`
maximum/minimum of x and y `max, min(x, y)`
integer or real factorial of x $x!$ or `factorial(x)`
derivative of f w.r.t. x f'

Conversions

Change Objects

to vector, matrix, set, list, string `Col/Vec, Mat, Set, List, Str`
create PARI object ($x \bmod y$) `Mod(x, y)`
make x a polynomial of v `Pol(x, {v})`
as above, starting with constant term `Polrev(x, {v})`
make x a power series of v `Ser(x, {v})`
PARI type of object x `type(x)`
object x with precision n `prec(x, {n})`
evaluate f replacing vars by their value `eval(f)`

Select Pieces of an Object

length of x `#x` or `length(x)`
 n -th component of x `component(x, n)`
 n -th component of vector/list x `x[n]`
 (m, n) -th component of matrix x `x[m, n]`
row m or column n of matrix x `x[m,], x[, n]`
numerator of x `numerator(x)`
lowest denominator of x `denominator(x)`

Conjugates and Lifts

conjugate of a number x `conj(x)`
conjugate vector of algebraic number x `conjvec(x)`
norm of x , product with conjugate `norm(x)`
square of L^2 norm of vector x `norml2(x)`
lift of x from Mods `lift, centerlift(x)`

Random Numbers

random integer between 0 and $N - 1$ `random({N})`
get random seed `getrand()`
set random seed to s `setrand(s)`

Lists, Sets & Sorting

sort x by k th component `vecsort(x, {k}, {fl = 0})`
Sets (= row vector of strings with strictly increasing entries)
intersection of sets x and y `setintersect(x, y)`
set of elements in x not belonging to y `setminus(x, y)`
union of sets x and y `setunion(x, y)`
look if y belongs to the set x `setsearch(x, y, {flag})`
Lists
create empty list L `L = List()`
append x to list L `listput(L, x, {i})`
remove i -th component from list L `listpop(L, {i})`
insert x in list L at position i `listinsert(L, x, i)`
sort the list L in place `listsort(L, {flag})`

Programming & User Functions

Control Statements (X : formal parameter in expression seq)
eval. seq for $a \leq X \leq b$ `for(X = a, b, seq)`
eval. seq for X dividing n `fordiv(n, X, seq)`
eval. seq for primes $a \leq X \leq b$ `forprime(X = a, b, seq)`
eval. seq for $a \leq X \leq b$ stepping s `forstep(X = a, b, s, seq)`
multivariable for `forvec(X = v, seq)`
if $a \neq 0$, evaluate seq_1 , else seq_2 `if(a, {seq1}, {seq2})`
evaluate seq until $a \neq 0$ `until(a, seq)`
while $a \neq 0$, evaluate seq `while(a, seq)`
exit n innermost enclosing loops `break({n})`
start new iteration of n th enclosing loop `next({n})`
return x from current subroutine `return({x})`
error recovery (try seq_1) `trap({err}, {seq2}, {seq1})`

Input/Output

print args with/without newline `print(), print1()`
formatted printing `printf()`
read a string from keyboard `input()`
output $args$ in T_EX format `printtex(args)`
write $args$ to file `write, writel, writetex(file, args)`
read file into GP `read({file})`

Interface with User and System

allocates a new stack of s bytes `allocatemem({s})`
execute system command a `system(a)`
as above, feed result to GP `extern(a)`
install function from library `install(f, code, {gpf}, {lib})`
alias old to new `alias(new, old)`
new name of function f in GP 2.0 `whatnow(f)`

User Defined Functions

`name(formal vars) = my(local vars); seq`
`struct.member = seq`
kill value of variable or function x `kill(x)`

Iterations, Sums & Products

numerical integration `intnum(X = a, b, expr, {flag})`
sum $expr$ over divisors of n `sumdiv(n, X, expr)`
sum $X = a$ to $X = b$, initialized at x `sum(X = a, b, expr, {x})`
sum of series $expr$ `suminf(X = a, expr)`
sum of alternating/positive series `sumalt, sumpos`
product $a \leq X \leq b$, initialized at x `prod(X = a, b, expr, {x})`
product over primes $a \leq X \leq b$ `prodeuler(X = a, b, expr)`
infinite product $a \leq X \leq \infty$ `prodinf(X = a, expr)`
real root of $expr$ between a and b `solve(X = a, b, expr)`

Vectors & Matrices

| | |
|-----------------------------------|--|
| dimensions of matrix x | <code>matsize(x)</code> |
| concatenation of x and y | <code>concat($x, \{y\}$)</code> |
| extract components of x | <code>vecextract($x, y, \{z\}$)</code> |
| transpose of vector or matrix x | <code>mattranspose(x)</code> or <code>x-</code> |
| adjoint of the matrix x | <code>matadjoint(x)</code> |
| eigenvectors of matrix x | <code>mateigen(x)</code> |
| characteristic polynomial of x | <code>charpoly($x, \{v\}, \{flag\}$)</code> |
| minimal polynomial of x | <code>minpoly($x, \{v\}$)</code> |
| trace of matrix x | <code>trace(x)</code> |

Constructors & Special Matrices

| | |
|---|--|
| row vec. of $expr$ eval'd at $1 \leq i \leq n$ | <code>vector($n, \{i\}, \{expr\}$)</code> |
| col. vec. of $expr$ eval'd at $1 \leq i \leq n$ | <code>vectorv($n, \{i\}, \{expr\}$)</code> |
| matrix $1 \leq i \leq m, 1 \leq j \leq n$ | <code>matrix($m, n, \{i\}, \{j\}, \{expr\}$)</code> |
| diagonal matrix with diagonal x | <code>matdiagonal(x)</code> |
| $n \times n$ identity matrix | <code>matid(n)</code> |
| Hessenberg form of square matrix x | <code>mathess(x)</code> |
| $n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$ | <code>mathilbert(n)</code> |
| $n \times n$ Pascal triangle $P_{ij} = \binom{i}{j}$ | <code>matpascal($n - 1$)</code> |
| companion matrix to polynomial x | <code>matcompanion(x)</code> |

Gaussian elimination

| | |
|--|---|
| determinant of matrix x | <code>matdet($x, \{flag\}$)</code> |
| kernel of matrix x | <code>matker($x, \{flag\}$)</code> |
| intersection of column spaces of x and y | <code>matintersect(x, y)</code> |
| solve $M * X = B$ (M invertible) | <code>matsolve(M, B)</code> |
| as solve, modulo D (col. vector) | <code>matsolvemod(M, D, B)</code> |
| one sol of $M * X = B$ | <code>matinverseimage(M, B)</code> |
| basis for image of matrix x | <code>matimage(x)</code> |
| supplement columns of x to get basis | <code>mat supplement(x)</code> |
| rows, cols to extract invertible matrix | <code>matindexrank(x)</code> |
| rank of the matrix x | <code>matrank(x)</code> |

Lattices & Quadratic Forms

| | |
|--|--|
| upper triangular Hermite Normal Form | <code>mathnf(x)</code> |
| HNF of x where d is a multiple of $\det(x)$ | <code>mathnfmod(x, d)</code> |
| elementary divisors of x | <code>matsnf(x)</code> |
| LLL-algorithm applied to columns of x | <code>qflll($x, \{flag\}$)</code> |
| like qflll, x is Gram matrix of lattice | <code>qflllgram($x, \{flag\}$)</code> |
| LLL-reduced basis for kernel of x | <code>matkerint(x)</code> |
| \mathbf{Z} -lattice $\longleftrightarrow \mathbf{Q}$ -vector space | <code>matrixqz(x, p)</code> |
| signature of quad form $t^y * x * y$ | <code>qfsign(x)</code> |
| decomp into squares of $t^y * x * y$ | <code>qfgaussred(x)</code> |
| find up to m sols of $t^y * x * y \leq b$ | <code>qfminim(x, b, m)</code> |
| $v, v[i] :=$ number of sols of $t^y * x * y = i$ | <code>qfrep($x, B, \{flag\}$)</code> |
| eigenvals/eigenvecs for real symmetric x | <code>qfjacobi(x)</code> |

Formal & p-adic Series

| | |
|---|--|
| truncate power series or p -adic number | <code>truncate(x)</code> |
| valuation of x at p | <code>valuation(x, p)</code> |
| Dirichlet and Power Series | |
| Taylor expansion around 0 of f w.r.t. x | <code>taylor(f, x)</code> |
| $\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$ | <code>serconvol(x, y)</code> |
| $f = \sum a_k t^k$ from $\sum (a_k / k!) t^k$ | <code>serlaplace(f)</code> |
| reverse power series F so $F(f(x)) = x$ | <code>serreverse(f)</code> |
| Dirichlet series multiplication / division | <code>dirmul, dirdiv(x, y)</code> |
| Dirichlet Euler product (b terms) | <code>direuler($p = a, b, expr$)</code> |

p-adic Functions

| | |
|-------------------------------------|--|
| Teichmuller character of x | <code>teichmuller(x)</code> |
| Newton polygon of f for prime p | <code>newtonpoly(f, p)</code> |

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Polynomials & Rational Functions

| | |
|---|--|
| degree of f | <code>poldegree(f)</code> |
| coefficient of degree n of f | <code>polcoeff(f, n)</code> |
| round coeffs of f to nearest integer | <code>round($f, \{\&e\}$)</code> |
| gcd of coefficients of f | <code>content(f)</code> |
| replace x by y in f | <code>subst(f, x, y)</code> |
| discriminant of polynomial f | <code>poldisc(f)</code> |
| resultant of f and g | <code>polresultant($f, g, \{v\}, \{flag\}$)</code> |
| as above, give $[u, v, d], xu + yv = d$ | <code>bezoutres(x, y)</code> |
| derivative of f w.r.t. x | <code>deriv(f, x)</code> |
| formal integral of f w.r.t. x | <code>intformal(f, x)</code> |
| reciprocal poly $x^{\deg f} f(1/x)$ | <code>polrecip(f)</code> |
| interpol. pol. eval. at a | <code>polinterpolate($X, \{Y\}, \{a\}, \{\&e\}$)</code> |
| initialize t for Thue equation solver | <code>thueinit(f)</code> |
| solve Thue equation $f(x, y) = a$ | <code>thue($t, a, \{sol\}$)</code> |

Roots and Factorization

| | |
|--|--|
| number of real roots of $f, a < x \leq b$ | <code>polsturm($f, \{a\}, \{b\}$)</code> |
| complex roots of f | <code>polroots(f)</code> |
| symmetric powers of roots of f up to n | <code>polsym(f, n)</code> |
| roots of f mod p | <code>polrootsmod($f, p, \{flag\}$)</code> |
| factor f | <code>factor($f, \{lim\}$)</code> |
| factorization of f mod p | <code>factormod($f, p, \{flag\}$)</code> |
| factorization of f over \mathbf{F}_{p^a} | <code>factorff(f, p, a)</code> |
| p -adic fact. of f to prec. r | <code>factorpadic($f, p, r, \{flag\}$)</code> |
| p -adic roots of f to prec. r | <code>polrootspadic(f, p, r)</code> |
| p -adic root of f cong. to a mod p | <code>padicappr(f, a)</code> |
| Newton polygon of f for prime p | <code>newtonpoly(f, p)</code> |

Special Polynomials

| | |
|--|--|
| n th cyclotomic polynomial in var. v | <code>polcyclo($n, \{v\}$)</code> |
| d -th degree subfield of $\mathbf{Q}(\zeta_n)$ | <code>polsubcyclo($n, d, \{v\}$)</code> |
| n -th Legendre polynomial | <code>pollegendre($n, \{v = x\}$)</code> |
| n -th Tchebicheff polynomial | <code>polchebyshev($n, \{flag\}, \{v = x\}$)</code> |
| Zagier's polynomial of index n, m | <code>polzagier(n, m)</code> |

Transcendental Functions

| | |
|--|--|
| real, imaginary part of x | <code>real(x), imag(x)</code> |
| absolute value, argument of x | <code>abs(x), arg(x)</code> |
| square/ n th root of x | <code>sqrtn($x, n, \{\&z\}$)</code> |
| trig functions | <code>sin, cos, tan, cotan</code> |
| inverse trig functions | <code>asin, acos, atan</code> |
| hyperbolic functions | <code>sinh, cosh, tanh</code> |
| inverse hyperbolic functions | <code>asinh, acosh, atanh</code> |
| exponential of x | <code>exp(x)</code> |
| natural log of x | <code>ln(x)</code> or <code>log(x)</code> |
| gamma function $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$ | <code>gamma(x)</code> |
| logarithm of gamma function | <code>lngamma(x)</code> |
| $\psi(x) = \Gamma'(x) / \Gamma(x)$ | <code>psi(x)</code> |
| incomplete gamma function ($y = \Gamma(s)$) | <code>incgam($s, x, \{y\}$)</code> |
| exponential integral $\int_x^\infty e^{-t} / t dt$ | <code>eint1(x)</code> |
| error function $2 / \sqrt{\pi} \int_x^\infty e^{-t^2} dt$ | <code>erfc(x)</code> |
| dilogarithm of x | <code>dilog(x)</code> |
| m th polylogarithm of x | <code>polylog($m, x, \{flag\}$)</code> |
| U -confluent hypergeometric function | <code>hyperu(a, b, u)</code> |
| J -Bessel function, $J_{n+1/2}(x)$ | <code>besselj(n, x), besseljh(n, x)</code> |
| K -Bessel function of index nu | <code>besselk(nu, x)</code> |

Elementary Arithmetic Functions

| | |
|--|---|
| vector of binary digits of $ x $ | <code>binary(x)</code> |
| give bit number n of integer x | <code>bittest(x, n)</code> |
| ceiling of x | <code>ceil(x)</code> |
| floor of x | <code>floor(x)</code> |
| fractional part of x | <code>frac(x)</code> |
| round x to nearest integer | <code>round($x, \{\&e\}$)</code> |
| truncate x | <code>truncate($x, \{\&e\}$)</code> |
| gcd/LCM of x and y | <code>gcd(x, y), lcm(x, y)</code> |
| gcd of entries of a vector/matrix | <code>content(x)</code> |
| Primes and Factorization | |
| add primes in v to the prime table | <code>addprimes(v)</code> |
| the n th prime | <code>prime(n)</code> |
| vector of first n primes | <code>primes(n)</code> |
| smallest prime $\geq x$ | <code>nextprime(x)</code> |
| largest prime $\leq x$ | <code>precprime(x)</code> |
| factorization of x | <code>factor($x, \{lim\}$)</code> |
| reconstruct x from its factorization | <code>factorback($f, \{e\}$)</code> |

Divisors

| | |
|---|---|
| number of distinct prime divisors | <code>omega(x)</code> |
| number of prime divisors with mult | <code>bigomega(x)</code> |
| number of divisors of x | <code>numdiv(x)</code> |
| row vector of divisors of x | <code>divisors(x)</code> |
| sum of (k -th powers of) divisors of x | <code>sigma($x, \{k\}$)</code> |

Special Functions and Numbers

| | |
|--|--|
| binomial coefficient $\binom{x}{y}$ | <code>binomial(x, y)</code> |
| Bernoulli number B_n as real | <code>bernreal(n)</code> |
| Bernoulli vector B_0, B_2, \dots, B_{2n} | <code>bernvec(n)</code> |
| n th Fibonacci number | <code>fibonacci(n)</code> |
| number of partitions of n | <code>numbpart(n)</code> |
| Euler ϕ -function | <code>eulerphi(x)</code> |
| Möbius μ -function | <code>moebius(x)</code> |
| Hilbert symbol of x and y (at p) | <code>hilbert($x, y, \{p\}$)</code> |
| Kronecker-Legendre symbol $(\frac{x}{y})$ | <code>kronecker(x, y)</code> |

Miscellaneous

| | |
|--|--|
| integer or real factorial of x | <code>x!</code> or <code>fact(x)</code> |
| integer square root of x | <code>sqrntint(x)</code> |
| solve $z \equiv x$ and $z \equiv y$ | <code>chinese(x, y)</code> |
| minimal u, v so $xu + yv = \gcd(x, y)$ | <code>bezout(x, y)</code> |
| multiplicative order of x (intmod) (i=0) | <code>znorder($x, \{o\}$)</code> |
| primitive root mod prime power q | <code>znprimroot(q)</code> |
| structure of $(\mathbf{Z}/n\mathbf{Z})^*$ | <code>znstar(n)</code> |
| continued fraction of x | <code>contfrac($x, \{b\}, \{lmax\}$)</code> |
| last convergent of continued fraction x | <code>contfracpnqn(x)</code> |
| best rational approximation to x | <code>bestappr(x, k)</code> |

True-False Tests

| | |
|--|--|
| is x the disc. of a quadratic field? | <code>isfundamental(x)</code> |
| is x a prime? | <code>isprime(x)</code> |
| is x a strong pseudo-prime? | <code>ispseudoprime(x)</code> |
| is x square-free? | <code>issquarefree(x)</code> |
| is x a square? | <code>issquare($x, \{\&n\}$)</code> |
| is pol irreducible? | <code>polisirreducible(pol)</code> |

Based on an earlier version by Joseph H. Silverman
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PARI-GP Reference Card (2)

(PARI-GP version 2.5.0)

Elliptic Curves

Elliptic curve initially given by 5-tuple $E = [a_1, a_2, a_3, a_4, a_6]$. Points are $[x, y]$, the origin is $[0]$.

Initialize elliptic struct. ell , i.e create `ellinit($E, \{flag\}$)`

$a_1, a_2, a_3, a_4, a_6, b_2, b_4, b_6, b_8, c_4, c_6, disc, j$. This data can be recovered by typing `ell.a1, ..., ell.j`. If $flag$ omitted, also

• E defined over \mathbf{R}

| | |
|-----------------------------------|------------------------|
| x -coords. of points of order 2 | <code>ell.roots</code> |
| real and complex periods | <code>ell.omega</code> |
| associated quasi-periods | <code>ell.eta</code> |
| volume of complex lattice | <code>ell.area</code> |

• E defined over \mathbf{Q}_p , $|j|_p > 1$

| | |
|-------------------------------------|------------------------|
| x -coord. of unit 2 torsion point | <code>ell.roots</code> |
| Tate's $[u^2, u, q]$ | <code>ell.tate</code> |
| Mestre's w | <code>ell.w</code> |

change curve E using $v = [u, r, s, t]$

`ellchangecurve(ell, v)`

change point z using $v = [u, r, s, t]$

`ellchangept(z, v)`

add points $z_1 + z_2$

`elladd(ell, z_1, z_2)`

subtract points $z_1 - z_2$

`ellsub(ell, z_1, z_2)`

compute $n \cdot z$

`ellpow(ell, z, n)`

check if z is on E

`ellisoncurve(ell, z)`

order of torsion point z

`ellorder(ell, z)`

y -coordinates of point(s) for x

`ellordinate(ell, x)`

point $[\wp(z), \wp'(z)]$ corresp. to z

`ellztopoint(ell, z)`

complex z such that $p = [\wp(z), \wp'(z)]$

`ellpointtoz(ell, p)`

Curves over finite fields, Pairings

random point on E

`random(ell)`

structure $\mathbf{Z}/d_1\mathbf{Z} \times \mathbf{Z}/d_2\mathbf{Z}$ of $E(\mathbf{F}_p)$

`ellgroup(ell, p)`

Weil pairing of m -torsion pts x, y `ellweilpairing(ell, x, y, m)`

Tate pairing of x, y ; x m -torsion `elltatepairing(ell, x, y, m)`

Curves over \mathbf{Q} and the L -function

canonical bilinear form taken at z_1, z_2 `ellbil(ell, z_1, z_2)`

canonical height of z `ellheight($ell, z, \{flag\}$)`

height regulator matrix for pts in x `ellheightmatrix(ell, x)`

cond, min mod, Tamagawa num $[N, v, c]$ `ellglobalred(ell)`

Kodaira type of p -fiber of E `elllocalred(ell, p)`

minimal model of E/\mathbf{Q} `ellminimalmodel($ell, \{&v\}$)`

p th coeff a_p of L -function, p prime `ellap(ell, p)`

k th coeff a_k of L -function `ellak(ell, k)`

vector of first n a_k 's in L -function `ellan(ell, n)`

$L(E, s)$, set $A \approx 1$ `elllseries($ell, s, \{A\}$)`

order of vanishing at 1 `ellanalyticrank($ell, \{eps\}$)`

$L^{(r)}(E, 1)$ `ellLi(ell, r)`

root number for $L(E, \cdot)$ at p `ellrootno($ell, \{p\}$)`

torsion subgroup with generators `elltors(ell)`

modular parametrization of E `elltaniyama(ell)`

Elldata package, Cremona's database:

db code \leftrightarrow $[conductor, class, index]$ `ellconvertname(s)`

generators of Mordell-Weil group `ellgenerators(E)`

look up E in database `ellidentify(E)`

all curves matching criterion `ellsearch(N)`

loop over curves with cond. from a to b `forell(E, a, b, seq)`

Elliptic & Modular Functions

arithmetic-geometric mean

`agm(x, y)`

elliptic j -function $1/q + 744 + \dots$

`ellj(x)`

Weierstrass σ function

`ellsigma($ell, z, \{flag\}$)`

Weierstrass \wp function

`ellwp($ell, \{z\}, \{flag\}$)`

Weierstrass ζ function

`ellzeta(ell, z)`

modified Dedekind η func. $\prod(1 - q^n)$

`eta($x, \{flag\}$)`

Jacobi sine theta function

`theta(q, z)`

k -th derivative at $z=0$ of $\theta(q, z)$

`thetanulk(q, k)`

Weber's f functions

`weber($x, \{flag\}$)`

Riemann's zeta $\zeta(s) = \sum n^{-s}$

`zeta(s)`

Graphic Functions

crude graph of $expr$ between a and b `plot($X = a, b, expr$)`

High-resolution plot (immediate plot)

plot $expr$ between a and b `ploto($X = a, b, expr, \{flag\}, \{n\}$)`

plot points given by lists lx, ly `plotdraw($lx, ly, \{flag\}$)`

terminal dimensions

`plotsizes()`

Rectwindow functions

init window w , with size x, y

`plotinit(w, x, y)`

erase window w

`plotkill(w)`

copy w to w_2 with offset (dx, dy)

`plotcopy(w, w_2, dx, dy)`

scale coordinates in w

`plotscale(w, x_1, x_2, y_1, y_2)`

ploto in w `plotrecth($w, X = a, b, expr, \{flag\}, \{n\}$)`

plotdraw in w `plotrecthdraw($w, data, \{flag\}$)`

draw window w_1 at $(x_1, y_1), \dots$ `plotdraw($[[w_1, x_1, y_1], \dots]$)`

Low-level Rectwindow Functions

set current drawing color in w to c

`plotcolor(w, c)`

current position of cursor in w

`plotcursor(w)`

write s at cursor's position

`plotstring(w, s)`

move cursor to (x, y)

`plotmove(w, x, y)`

move cursor to $(x + dx, y + dy)$

`plotrmove(w, dx, dy)`

draw a box to (x_2, y_2)

`plotbox(w, x_2, y_2)`

draw a box to $(x + dx, y + dy)$

`plotrbox(w, dx, dy)`

draw polygon

`plotlines($w, lx, ly, \{flag\}$)`

draw points

`plotpoints(w, lx, ly)`

draw line to $(x + dx, y + dy)$

`plotrline(w, dx, dy)`

draw point $(x + dx, y + dy)$

`plotrpoint(w, dx, dy)`

Postscript Functions

as ploto `psploto($X = a, b, expr, \{flag\}, \{n\}$)`

as plotdraw `psplotdraw($lx, ly, \{flag\}$)`

as plotdraw `psdraw($[[w_1, x_1, y_1], \dots]$)`

Binary Quadratic Forms

create $ax^2 + bxy + cy^2$ (distance d) `qfb($a, b, c, \{d\}$)`

reduce x ($s = \sqrt{D}$, $l = \lfloor s \rfloor$) `qfbred($x, \{flag\}, \{D\}, \{l\}, \{s\}$)`

composition of forms $x*y$ or `qfbnucomp(x, y, l)`

n -th power of form x^n or `qfbnpow(x, n)`

composition without reduction `qfbcomprow(x, y)`

n -th power without reduction `qfbpowrow(x, n)`

prime form of disc. x above prime p `qfbprimeform(x, p)`

class number of disc. x `qfbclassno(x)`

Hurwitz class number of disc. x `qfbhclassno(x)`

Quadratic Fields

quadratic number $\omega = \sqrt{x}$ or $(1 + \sqrt{x})/2$ `quadgen(x)`

minimal polynomial of ω `quadpoly(x)`

discriminant of $\mathbf{Q}(\sqrt{D})$ `quaddisc(x)`

regulator of real quadratic field `quadregulator(x)`

fundamental unit in real $\mathbf{Q}(x)$ `quadunit(x)`

class group of $\mathbf{Q}(\sqrt{D})$ `quadclassunit($D, \{flag\}, \{t\}$)`

Hilbert class field of $\mathbf{Q}(\sqrt{D})$ `quadhilbert($D, \{flag\}$)`

ray class field modulo f of $\mathbf{Q}(\sqrt{D})$ `quadray($D, f, \{flag\}$)`

General Number Fields: Initializations

A number field K is given by a monic irreducible $f \in \mathbf{Z}[X]$.

init number field structure nf `nfinit($f, \{flag\}$)`

nf members:

polynomial defining nf , $f(\theta) = 0$ `nf.pol`

number of real/complex places `nf.r1/r2/sign`

discriminant of nf `nf.disc`

T_2 matrix `nf.t2`

vector of roots of f `nf.roots`

integral basis of \mathbf{Z}_K as powers of θ `nf.zk`

different `nf.diff`

codifferent `nf.codiff`

index `nf.index`

recompute nf using current precision `nfnewprec(nf)`

init relative rnf given by $g = 0$ over K `rnfinit(nf, g)`

init bnf structure `bnfinit($f, \{flag\}$)`

bnf members: same as nf , plus

underlying nf `bnf.nf`

classgroup `bnf.clgp`

regulator `bnf.reg`

fundamental units `bnf.fu`

torsion units `bnf.tu`

compute a bnf from small bnf `bnfinit($sbnf$)`

add S -class group and units, yield bnf s `bnfsunit(nf, S)`

init class field structure bnr `bnrinit($bnf, m, \{flag\}$)`

bnr members: same as bnf , plus

underlying bnf `bnr.bnf`

big ideal structure `bnr.bid`

modulus `bnr.mod`

structure of $(\mathbf{Z}_K/m)^*$ `bnr.zkst`

Basic Number Field Arithmetic (nf)

Elements are `t_INT`, `t_FRAC`, `t_POL`, `t_POLMOD`, or `t_COL` (on integral basis `nf.zk`). Basic operations (prefix `nfelt`): `(nfelt)add`, `mul`, `pow`, `div`, `diveuc`, `mod`, `divrem`, `val`, `trace`, `norm`
express x on integer basis `nfalgtobasis(nf, x)`
express element x as a polmod `nfbasistoalg(nf, x)`
reverse polmod $a = A(X) \bmod T(X)$ `modreverse(a)`
integral basis of field def. by $f = 0$ `nfbasis(f)`
field discriminant of field $f = 0$ `nfdisc(f)`
Galois group of field $f = 0$, $\deg f \leq 11$ `polgalois(f)`
smallest poly defining $f = 0$ `polredabs(f, {flag})`
small polys defining subfields of $f = 0$ `polred(f, {flag}, {p})`
poly of degree $\leq k$ with root $x \in \mathbf{C}$ `algdep(x, k)`
small linear rel. on coords of vector x `lindep(x)`
are fields $f = 0$ and $g = 0$ isomorphic? `nfisom(f, g)`
is field $f = 0$ a subfield of $g = 0$? `nfisincl(f, g)`
compositum of $f = 0$, $g = 0$ `polcompositum(f, g, {flag})`
subfields (of degree d) of nf `nfsubfields(nf, {d})`
roots of unity in nf `nfrootsof1(nf)`
roots of g belonging to nf `nfroots({nf}, g)`
factor g in nf `nnffactor(nf, g)`
factor g mod prime pr in nf `nnffactormod(nf, g, pr)`
conjugates of a root θ of nf `nfgaloisconj(nf, {flag})`
apply Galois automorphism s to x `nfgaloisapply(nf, s, x)`
quadratic Hilbert symbol (at p) `nfhilbert(nf, a, b, {p})`
Dedekind Zeta Function ζ_K
 ζ_K as Dirichlet series, $N(I) < b$ `dirzetak(nf, b)`
init nfz for field $f = 0$ `zetakinit(f)`
compute $\zeta_K(s)$ `zetak(nfz, s, {flag})`
Artin root number of K `bnrrootnumber(bnr, chi, {flag})`

Class Groups & Units (bnf, bnr)

$a_1, \{a_2\}, \{a_3\}$ usually bnr , $subgp$ or bnf , $module$, $\{subgp\}$
remove GRH assumption from bnf `bnfcertify(bnf)`
expo. of ideal x on class gp `bnfisprincipal(bnf, x, {flag})`
expo. of ideal x on ray class gp `bnrisprincipal(bnr, x, {flag})`
expo. of x on fund. units `bnfisunit(bnf, x)`
as above for S -units `bnfissunit(bnfs, x)`
signs of real embeddings of bnf .fu `bnfsignunit(bnf)`

Class Field Theory

ray class number for mod. m `bnrclassno(bnf, m)`
discriminant of class field ext `bnrdisc(a1, {a2}, {a3})`
ray class numbers, l list of mods `bnrclassnolist(bnf, l)`
discriminants of class fields `bnrdisclist(bnf, l, {arch}, {flag})`
decode output from `bnrdisc` `bnfdecodemodule(nf, fa)`
is modulus the conductor? `bnrisconductor(a1, {a2}, {a3})`
conductor of character chi `bnrconductorofchar(bnr, chi)`
conductor of extension `bnrconductor(a1, {a2}, {a3}, {flag})`
conductor of extension def. by g `rnfconductor(bnf, g)`
Artin group of ext. def'd by g `rnfnormgroup(bnr, g)`
subgroups of bnr , index $\leq b$ `subgrouplist(bnr, b, {flag})`
rel. eq. for class field def'd by sub `rnfkummer(bnr, sub, {d})`
same, using Stark units (real field) `bnrstark(bnr, sub, {flag})`

PARI-GP Reference Card (2)

(PARI-GP version 2.5.0)

Ideals

Ideals are elements, primes, or matrix of generators in HNF.
is id an ideal in nf ? `nfisideal(nf, id)`
is x principal in bnf ? `bnfisprincipal(bnf, x)`
principal ideal generated by x `idealprincipal(nf, x)`
principal idele generated by x `ideleprincipal(nf, x)`
give $[a, b]$, s.t. $a\mathbf{Z}_K + b\mathbf{Z}_K = x$ `idealtwoelt(nf, x, {a})`
put ideal a ($a\mathbf{Z}_K + b\mathbf{Z}_K$) in HNF form `idealhnf(nf, a, {b})`
norm of ideal x `idealnrm(nf, x)`
minimum of ideal x (direction v) `idealmin(nf, x, v)`
LLL-reduce the ideal x (direction v) `idealred(nf, x, {v})`

Ideal Operations

add ideals x and y `idealadd(nf, x, y)`
multiply ideals x and y `idealmul(nf, x, y, {flag})`
intersection of ideals x and y `idealintersect(nf, x, y, {flag})`
 n -th power of ideal x `idealpow(nf, x, n, {flag})`
inverse of ideal x `idealinv(nf, x)`
divide ideal x by y `idealdiv(nf, x, y, {flag})`
Find $(a, b) \in x \times y$, $a + b = 1$ `idealaddtoone(nf, x, {y})`

Primes and Multiplicative Structure

factor ideal x in nf `idealfactor(nf, x)`
expand ideal factorization in nf `idealfactorback(nf, f, e)`
decomposition of prime p in nf `idealprimedec(nf, p)`
valuation of x at prime ideal pr `idealval(nf, x, pr)`
weak approximation theorem in nf `idealchinese(nf, x, y)`
give bid = structure of $(\mathbf{Z}_K/id)^*$ `idealstar(nf, id, {flag})`
discrete log of x in $(\mathbf{Z}_K/bid)^*$ `ideallog(nf, x, bid)`
idealstar of all ideals of norm $\leq b$ `ideallist(nf, b, {flag})`
add Archimedean places `ideallistarch(nf, b, {ar}, {flag})`
init `prmod` structure `nfmodprinit(nf, pr)`
kernel of matrix M in $(\mathbf{Z}_K/pr)^*$ `nfkermodpr(nf, M, prmod)`
solve $Mx = B$ in $(\mathbf{Z}_K/pr)^*$ `nfsolvemodpr(nf, M, B, prmod)`

Galois theory over \mathbf{q}

initializes a Galois group structure `galoisinit(pol, {den})`
action of p in `nfgaloisconj` form `galoispermopol(G, {p})`
identifies as abstract group `galoisidentify(G)`
exports a group for GAP or MAGMA `galoisexport(G, {flag})`
subgroups of the Galois group G `galoissubgroups(G)`
subfields from subgroups of G `galoissubfields(G, {flag}, {v})`
fixed field `galoisfixedfield(G, perm, {flag}, {v})`
is G abelian? `galoisisabelian(G, {flag})`
abelian number fields `galoissubcyclo(N, H, {flag}, {v})`

Relative Number Fields (rnf)

Extension L/K is defined by $g \in K[x]$. We have $order \subset L$.
absolute equation of L `rnfequation(nf, g, {flag})`
relative `nfalgtobasis` `rnfalgtobasis(rnf, x)`
relative `nfbasistoalg` `rnfbasistoalg(rnf, x)`
relative `idealhnf` `rnfidealhnf(rnf, x)`
relative `idealmul` `rnfidealmul(rnf, x, y)`
relative `idealtwoelt` `rnfidealtwoelt(rnf, x)`

Lifts and Push-downs

absolute \rightarrow relative repres. for x `rnfeltabstorel(rnf, x)`
relative \rightarrow absolute repres. for x `rnfeltreltoabs(rnf, x)`
lift x to the relative field `rnfeltup(rnf, x)`
push x down to the base field `rnfeltdown(rnf, x)`
idem for x ideal: `(rnfideal)reltoabs, astorel, up, down`

Projective \mathbf{Z}_K -modules, maximal order

relative `polred` `rnfpolred(nf, g)`
relative `polredabs` `rnfpolredabs(nf, g)`
characteristic poly. of $a \bmod g$ `rnfcharpoly(nf, g, a, {v})`
relative Dedekind criterion, prime pr `rnfdedekind(nf, g, pr)`
discriminant of relative extension `rnfdisc(nf, g)`
pseudo-basis of \mathbf{Z}_L `rnfpseudobasis(nf, g)`
relative HNF basis of $order$ `rnfhnfbasis(bnf, order)`
reduced basis for $order$ `rnflllgram(nf, g, order)`
determinant of pseudo-matrix A `rnfdet(nf, A)`
Steinitz class of $order$ `rnfsteynitz(nf, order)`
is $order$ a free \mathbf{Z}_K -module? `rnfisfree(bnf, order)`
true basis of $order$, if it is free `rnfbasis(bnf, order)`

Norms

absolute norm of ideal x `rnfidealnrmabs(rnf, x)`
relative norm of ideal x `rnfidealnrmrel(rnf, x)`
solutions of $N_K/\mathbf{Q}(y) = x \in \mathbf{Z}$ `bnfisintnorm(bnf, x)`
is $x \in \mathbf{Q}$ a norm from K ? `bnfisnorm(bnf, x, {flag})`
initialize T for norm eq. solver `rnfisnorminit(K, pol, {flag})`
is $a \in K$ a norm from L ? `rnfisnorm(T, a, {flag})`

Based on an earlier version by Joseph H. Silverman
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